Principle of virtual work :-

If the system of particles goes from one configuration to another, each particle of the system is imagined to be displaced by an infinitesimal vector $\delta\vec{r_i}$ from the old to new position. The displacement $\delta\vec{r_i}$ is called the virtual displacement.

In equilibrium, the resultant force acting on each particle of the system must be zero. i.e. $\vec{F_i} = 0$

Thus virtual work i.e, the product of force $\vec{F_i}$ on ith particle and its virtual displacement must be zero.

i.e, $\vec{F_i} \cdot \delta\vec{r_i} = 0$.

Summing up all such particles of the system,

$$\sum_i \vec{F_i} \cdot \delta\vec{r_i} = 0 \longrightarrow (1)$$

If the constraints are present, then -

$\vec{F_i}$ = constraint force + applied force.

$$= \vec{f_i} + \vec{F_i}^a \longrightarrow (2)$$

From equation (1) and (2), we get -

$$\sum_i (\vec{f_i} + \vec{F_i}^a) \cdot \delta\vec{r_i} = 0$$

$$\Rightarrow \sum_i \vec{f_i} \cdot \delta\vec{r_i} + \sum_i \vec{F_i}^a \cdot \delta\vec{r_i} = 0 \longrightarrow (3)$$

For a system where the virtual workdone by the forces of constraints is zero, i.e, $\sum_i \vec{f_i} \cdot \delta\vec{r_i} = 0$

Now, eqn (3) reduces to, $\boxed{\sum_i \vec{F_i}^a \cdot \delta\vec{r_i} = 0}$

" Thus a system of particles is in eqn only if the total virtual work of the actual or applied force is zero" $\longrightarrow$ this is known as principle of virtual work.